Proof of Sequent: Steps and Structure

Outstanding Sequent to Prove

$$d \in \mathbb{N}$$
$$n \in \mathbb{N}$$
$$n \leq d$$
$$\vdash$$
$$n + 1 \in \mathbb{N}$$

ML_out/inv0_1/INV

Known Inference Rules

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \textbf{MON}$$

$$\frac{}{n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}}\ \textbf{P2}$$

Understanding Inference Rule: OR_L

$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \vee Q \vdash R}\ \textbf{OR\_L}$$

Example Inference Rules

$$\frac{}{\vdash 0 \in \mathbb{N}}\ \textbf{P1}$$

$$\frac{}{n \in \mathbb{N} \vdash n+1 \in \mathbb{N}}\ \textbf{P2}$$

$$\frac{}{n < m \vdash n+1 \leq m}\ \textbf{INC}$$

$$\frac{}{0 < n \vdash n-1 \in \mathbb{N}}\ \textbf{P2'}$$

$$\frac{}{n \leq m \vdash n-1 < m}\ \textbf{DEC}$$

$$\frac{}{n \in \mathbb{N} \vdash 0 \leq n}\ \textbf{P3}$$

$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \vee Q \vdash R}\ \textbf{OR\_L}$$

$$\frac{H \vdash P}{H \vdash P \vee Q}\ \textbf{OR\_R1}$$

$$\frac{H \vdash Q}{H \vdash P \vee Q}\ \textbf{OR\_R2}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \textbf{MON}$$

Discharging POs of original m0: Invariant Preservation

ML_out/inv0_1/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n+1 \in \mathbb{N}$

ML_in/inv0_1/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n-1 \in \mathbb{N}$

ML_out/inv0_2/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n+1 \leq d$

ML_in/inv0_2/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n-1 \leq d$

$$\frac{H \vdash P}{H \vdash P \vee Q}\ \text{OR\_R1}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON}$$

$$\frac{}{n \leq m \vdash n-1 < m}\ \text{DEC}$$

$$\frac{}{n \in \mathbb{N} \vdash n+1 \in \mathbb{N}}\ \text{P2}$$